Regularization of inverse source problem for hyperbolic equations

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Inverse source problems for hyperbolic equations $Aq=f$ are ill-posed problems, *i.e.* their solutions are not unique or/and unstable. These problems should be regularized. In this talk we apply regularization techniques and control the degree of ill-posedness of several methods, such as Fourier series expansion, truncated singular value decomposition of compact operator A and its discrete analogue, iterative regularization. An effective tool in regularization of an unstable inverse problem is some additional information about the source: additional measurements (combined data), mathematical properties of the source, *etc*.

As an example we consider combined inverse source problem for the linear shallow water equations. We investigate two different inverse problems of determining a tsunami source using two different additional data: measurements of the height of a passing tsunami wave at several given points of the coastal area and measurements of the wave distribution at a fixed time. We consider the operator form of each inverse problem and describe the algorithm of selecting the truncated number of singular values of each inverse problem operator which is agreed with the error level in measured data. We reduce each inverse problem $Aq=f$ to the problem of minimization the cost functional $J(q)=||Aq-f||^2$. To calculate the gradient of the misfit functions, the adjoint problems are solved. In numerical experiment we used gradient methods (Landweber iteration and conjugate gradient method) for solving inverse source problems.

The main idea consists of combination of two measured data to reconstruct the source parameters. Results of numerical experiments of the tsunami source reconstruction are presented and discussed. We show that using a combination of two types of data allows one to increase the stability and efficiency of source reconstruction.

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